

Cooperative Networked Radar: The Two-Step Detector

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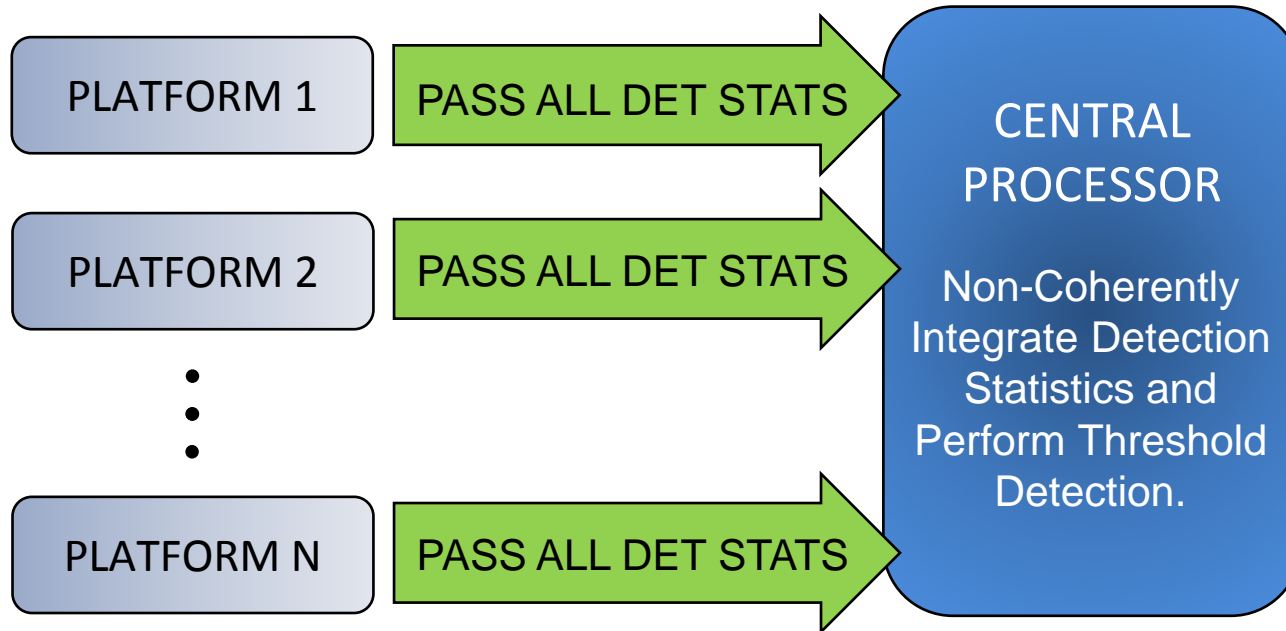
Outline

- **Background and Motivation: Distributed Detection**
- Neyman-Pearson (NP) Two-Step Detection Rule
- Analysis of Performance
- Performance Results
- Conclusions

Background and Motivation: Distributed Detection

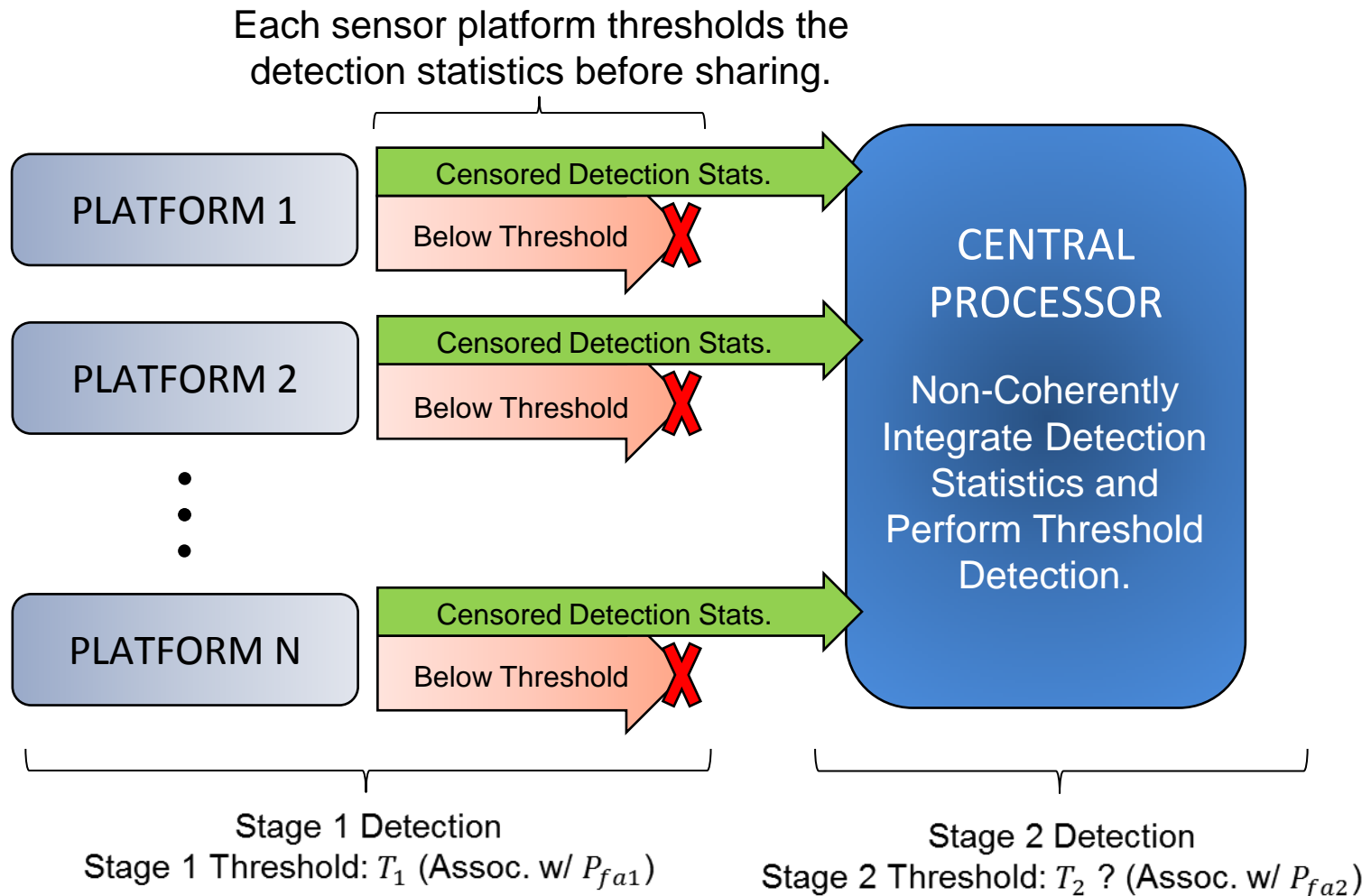
- The study of the two-step detector is motivated by a practical implementation problem in distributed sensing/detection.
- The integration of raw/pre-track data from multiple sensor platforms enables the detection of stealthy targets which are undetectable with a single platform.
- The sharing of raw/pre-track data will require more communications bandwidth.
- What do we do if there are practical limits on the amount of data that can be shared?
- The two-step detection scheme addresses this problem.

Distributed Detection: Unrestricted Case



- One approach to reducing the aggregate data rate is to initially threshold (or censor) the detection statistics at each platform prior to sharing.
- Assuming the number of targets is small compared to the number of detection cells most of the detection statistics are due to noise.
- On each platform if we pass the detection statistics through an initial detector with a threshold associated with probability of false alarm (P_{fa}), we should reduce the data rate by a factor of $\sim P_{fa}$.
- Aggregate data rate is directly linked with the initial detector's P_{fa} .

Distributed Detection: The Two-Step Detection Scheme

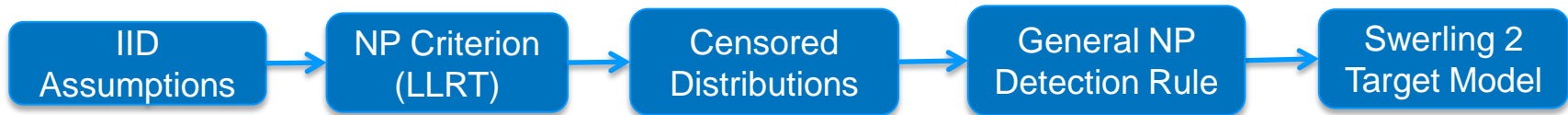


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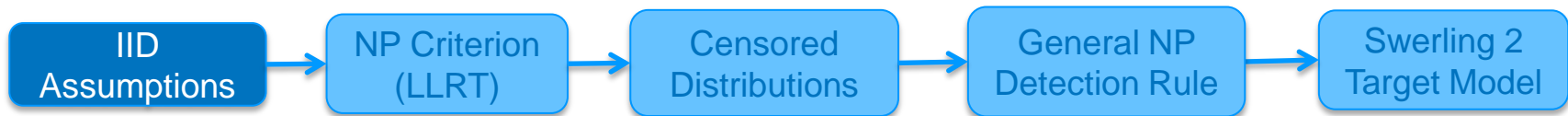
NP Two-Step Detection Rule: Roadmap

- In the following slides we derive the NP detection rule for the two-step detection scheme (2SD) using the Neyman-Pearson (NP) criterion.
- To make the problem tractable we make IID assumptions.
- Based on assumptions and NP criterion use the LLRT and
- Properties of censored distributions to
- Express the general NP two-step detection rule.
- Obtain the NP two-step detection rule for Swerling 2 (fluctuating RCS) target model.



NP Two-Step Detection Rule: Assumptions

- Assumptions:
 - Noise at each platform is IID Gaussian I/Q.
 - The target SNR measured by each platform is an IID random variable.
 - The detection cells (e.g., range-gates) for each platform align exactly (i.e., there are no registration errors).



NP Two-Step Detection Rule: NP Criterion

- Based on the NP criterion, the general form of the NP detection rule is (\mathcal{H}_1 = target present, \mathcal{H}_0 = noise only):

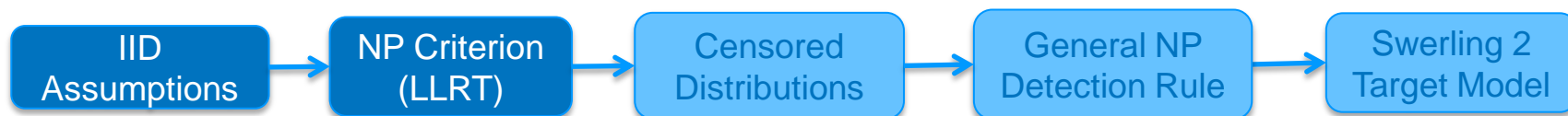
$$\Lambda(x_1, \dots, x_N) = \frac{\hat{p}(x_1, \dots, x_N | \mathcal{H}_1)}{\hat{p}(x_1, \dots, x_N | \mathcal{H}_0)} > \lambda \rightarrow \mathcal{H}_1$$

- By assumptions (i-iii) the joint probability densities can be factored into the product of marginal densities:

$$\hat{p}(x_1, \dots, x_N | \mathcal{H}_i) = \prod_{k=1}^N \hat{p}(x_k | \mathcal{H}_i)$$

- Taking the $\ln(\cdot)$ (LLRT):

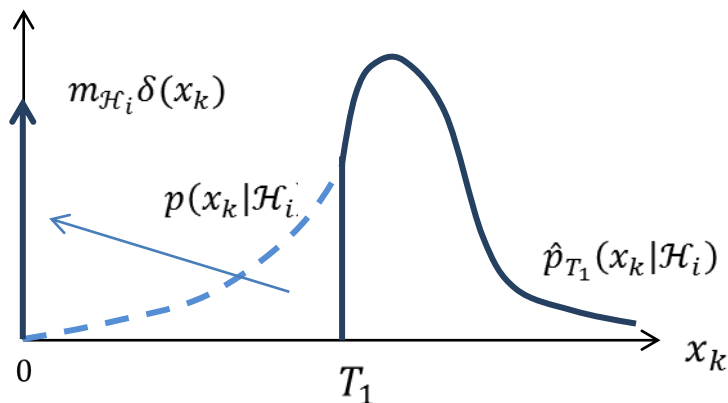
$$\sum_{k=1}^N \{ \ln(\hat{p}(x_k | \mathcal{H}_1)) - \ln(\hat{p}(x_k | \mathcal{H}_0)) \} \stackrel{\mathcal{H}_1}{>} \ln(\lambda)$$



NP Two-Step Detection Rule: Censored Distributions

- After Stage 1 thresholding the detection statistics are distributed as censored versions of their original distributions:

$$\hat{p}(x_k|\mathcal{H}_i; T_1) = p(x_k|\mathcal{H}_i)u(x_k - T_1) + m_{\mathcal{H}_i}^k \delta(x_k)$$

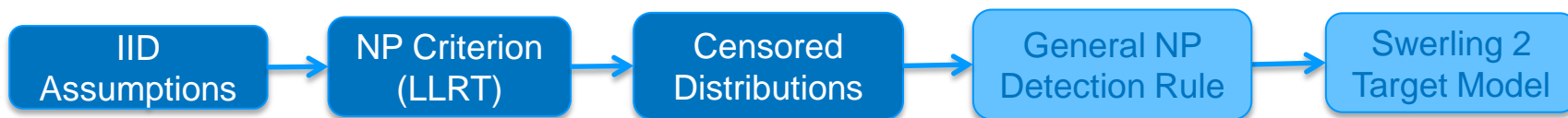


- In the case of \mathcal{H}_0 :

$$m_{\mathcal{H}_0}^k = \int_0^{T_1} p(x_k|\mathcal{H}_0) dx_k = 1 - P_{fa1}$$

- In the case of \mathcal{H}_1 :

$$m_{\mathcal{H}_1}^k = \int_0^{T_1} p(x_k|\mathcal{H}_1) dx_k = 1 - P_{d1}$$



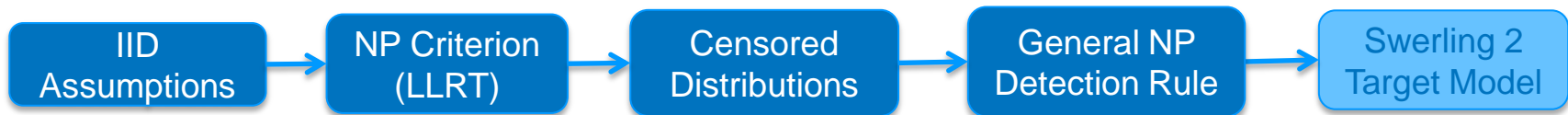
NP Two-Step Detection Rule: General Rule

- Thus the general NP two-step detection rule is decide \mathcal{H}_1 if:

$$\sum_{x_k \geq T_1} \{ \ln(p(x_k | \mathcal{H}_0)) - \ln(p(x_k | \mathcal{H}_1)) \} + (N - L) \ln \left(\frac{1 - P_{d1}}{1 - P_{fa1}} \right)^{\mathcal{H}_1} > \ln(\lambda)$$

- L (detection level) is the number of x_k s greater than T_1 :

$$L = \sum_{x_k \geq T_1} 1$$



NP Two-Step Detection Rule: Swerling 2 Target Model

- The SW2 detection statistic, x_k , where M pulses are non-coherently integrated with a given average target SNR, ξ , has the following PDFs under \mathcal{H}_0 and \mathcal{H}_1 :

$$p(x_k|\mathcal{H}_0) = \frac{x_k^{(M-1)}}{(M-1)!} e^{-x_k} \quad p(x_k|\mathcal{H}_1) = \frac{x_k^{(M-1)}}{(1+\xi)^M (M-1)!} e^{-x_k/(1+\xi)}$$

- Plugging these pdfs into the general rule:

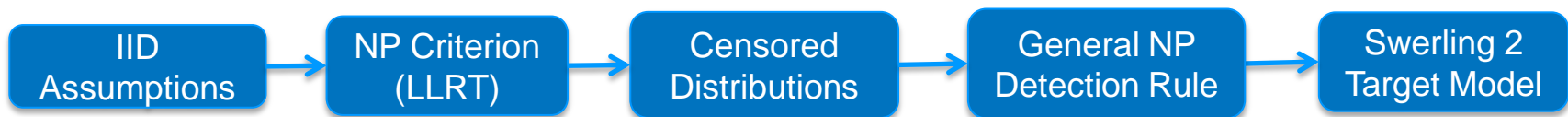
$$\sum_{x_k \geq T_1} \left\{ \frac{\xi}{(1+\xi)} x_k - M \ln(1+\xi) \right\} + (N-L) \ln \left(\frac{1-P_{d1}}{1-P_{fa1}} \right) \stackrel{\mathcal{H}_1}{>} \ln(\lambda)$$

- After some simplifications:

$$\sum_{x_k \geq T_1} x_k \stackrel{\mathcal{H}_1}{>} \eta + L\beta(\xi) = T_{2,L}(\xi)$$

- Where:

$$\eta = \left(\ln(\lambda) - N \ln \left(\frac{1-P_{d1}}{1-P_{fa1}} \right) \right) \quad \beta(\xi) = \frac{(1+\xi)}{\xi} \left(M \ln(1+\xi) + \ln \left(\frac{1-P_{d1}}{1-P_{fa1}} \right) \right)$$



The Swerling 2 NP Two-Step Detection Rule: Observations

$$\sum_{x_k \geq T_1} x_k \stackrel{\mathcal{H}_1}{>} \eta + L\beta(\xi) = T_{2,L}(\xi)$$

(ii) (iii) (i)

Observations:

- i. The NP detector is a clairvoyant detector since it is depended on an unknown parameter ξ , the average target SNR.
- ii. The detection rule involves a linear combination of the shared detection statistics.
- iii. Assuming knowledge of ξ there is only a single threshold parameter, η , that needs to be found based on the desired overall probability of false alarm (P_{fa2}).
- iv. It is straightforward to see that as $T_1 \rightarrow 0$, the two-step detection rule becomes the familiar single-stage NP detector for the SW2 case (square-law detector):

$$\sum_{L=1}^N x_k \stackrel{\mathcal{H}_1}{>} \eta'$$

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- **Analysis of Performance**
 - **Second-Stage Probability of False-Alarm**
 - **Threshold Selection**
 - **Second-Stage Probability of Detection**
- Performance Results
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Analysis of Performance: Second-Stage Probability of False Alarm (1 of 2)

- To compute the threshold parameter for the two-step detector we have derived the following general expression for the second-stage probability of false alarm:

$$P_{fa2} = \sum_{L=1}^N \binom{N}{L} (1 - P_{fa1})^{(N-L)} (P_{fa1})^L \Phi_L(T_{2,L})$$

- Where:

$$\Phi_L(T_{2,L}) \stackrel{\text{def}}{=} \Pr \left(\sum_{l=1}^L X_{(l)} > T_{2,L} \mid X_{(1)} > T_1, \dots, X_{(L)} > T_1, \mathcal{H}_0 \right)$$

- Φ_L is the complimentary CDF of the second-stage detection statistic given that L of the first-stage detection statistics are greater than T_1 and no target is present (\mathcal{H}_0).
- The second-stage detection statistic is the sum of L left-truncated IID random variables. Thus, the PDF corresponding to Φ_L is an L -fold convolution of left-truncated PDFs.

Analysis of Performance: Second-Stage Probability of False Alarm (2 of 2)

- We have derived a closed-form expression for $\Phi_L(T_{2,L})$ that turns out to be a mixture of Gamma densities:

$$\Phi_L(T_{2,L}) = \sum_{r=0}^{L(M-1)} b_r \Gamma(T_{2,L}, LM - r)$$

- Where b_r is the r^{th} coefficient resulting from the L -fold convolution:

$$\mathbf{b} = \mathbf{a} * \mathbf{a} * \dots * \mathbf{a} = \mathbf{a}^{L*}$$

- Of the a_k coefficients:

$$a_k = \frac{e^{-T_1} T_1^k}{P_{fa1} k!}$$

- Now we can select the thresholds.

Analysis of Performance: Threshold Selection

- Recall:

$$\sum_{x_k \geq T_1} x_k > \eta + L\beta(\xi) = T_{2,L}(\xi)$$

- Assuming knowledge of the target SNR, ξ , each detection level threshold, $T_{2,L}$, is a function of a single parameter η .

$$P_{fa2} = \sum_{L=1}^N \binom{N}{L} (1 - P_{fa1})^{(N-L)} (P_{fa1})^L \Phi_L(T_{2,L})$$

- It is straightforward to show that P_{fa2} is monotonic in η , therefore η can be found using a simple root-finding method and subsequently each $T_{2,L}$ can be easily derived.

Analysis of Performance: Second-Stage Probability of Detection (1 of 2)

- To compute the threshold parameter for the two-step detector we have derived the following general expression for the second-stage probability of detection:

$$P_{d2} = \sum_{L=1}^N \binom{N}{L} (1 - P_{d1})^{(N-L)} (P_{d1})^L F_L(T_{2,L})$$

- Where:

$$F_L(T_{2,L}) \stackrel{\text{def}}{=} \Pr \left(\sum_{l=1}^L X_{(l)} > T_{2,L} \mid X_{(1)} > T_1, \dots, X_{(L)} > T_1, \mathcal{H}_1 \right)$$

- F_L is the complimentary CDF of the second-stage detection statistic given that L of the first-stage detection statistics are greater than T_1 and a target is present (\mathcal{H}_1).
- The second-stage detection statistic is the sum of L left-truncated IID random variables. Thus, the PDF corresponding to F_L is an L -fold convolution of left-truncated PDFs.

Analysis of Performance: Second-Stage Probability of Detection (2 of 2)

- We have derived a closed-form expression for $F_L(T_{2,L})$ that turns out to be a mixture of Gamma densities:

$$F_L(T_{2,L}) = \sum_{r=0}^{L(M-1)} d_r \Gamma\left(\frac{T_{2,L}}{(1+\xi)}, LM - r\right)$$

- Where d_r is the r^{th} coefficient resulting from the L -fold convolution:

$$\mathbf{d} = \mathbf{c} * \mathbf{c} * \dots * \mathbf{c} = \mathbf{c}^{L*}$$

- Of the c_k coefficients:

$$c_k = \frac{e^{-\frac{T_1}{1+\xi}} T_1^k}{P_{d1} (1+\xi)^k k!}$$

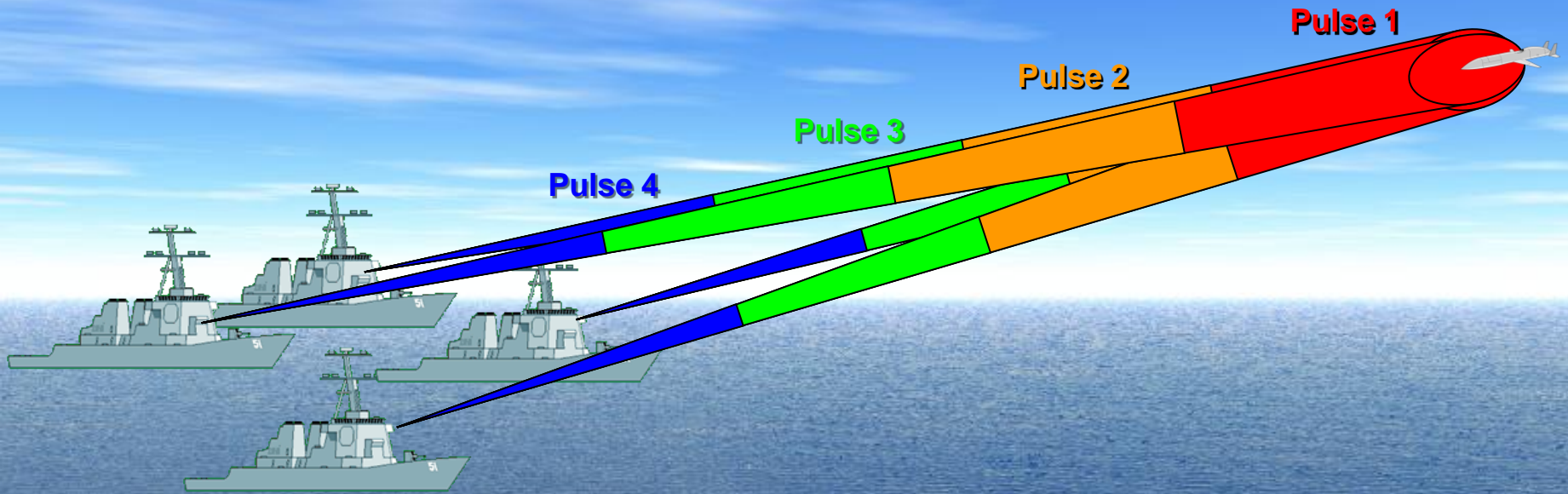
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Results: Cooperative Networked Radar (CNR) Scenario

- CNR is a multi-static/MIMO radar scheme.
- Exploits the presence of multiple ships to increase both the coherent and non-coherent integration used:
 - Instead of each ship transmitting multiple pulses on multiple frequencies, each ship only transmits a single longer pulse on just one frequency (coherent gain).
 - The ships receive and process their own pulse plus those of the other participating vessels.
 - Pre-detection data is non-coherently integrated for improved sensitivity (non-coherent gain).
- A practical implementation issue with CNR is that the cooperative data rate can saturate both the computational and communication capacity of the system.

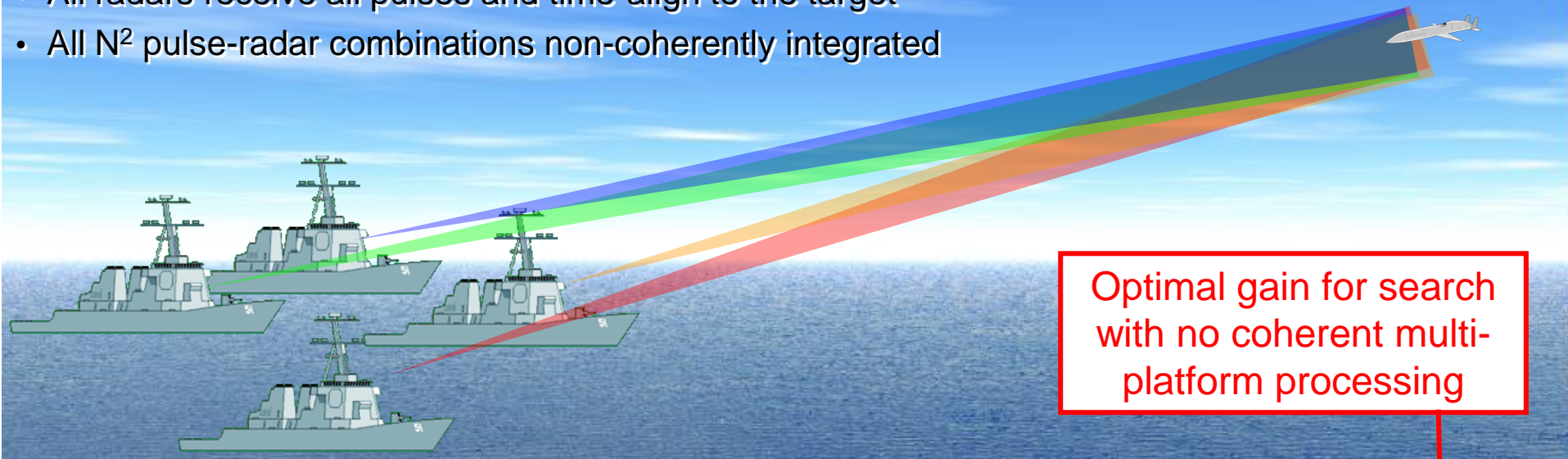
Current Processing Scheme (Non Cooperative)



- Radars operate independently
 - Hopefully they do not interfere with each other!
- Performance limited by each radar's mono-static performance
- Each radar non-coherently integrates a few (e.g. 4) pulses

Cooperative Radar Description & Gain

- Each radar transmits a single pulse 4x longer than the mono-static case
- Each radar transmits on a different frequency
- Radars transmit at approximately the same time
- All radars receive all pulses and time-align to the target
- All N^2 pulse-radar combinations non-coherently integrated



Optimal gain for search with no coherent multi-platform processing

Potential Cooperative Radar Gain (All Radars Same Distance to Target)

No. Radars	Coherent Gain	Non Coh. Gain SW0	Total Gain SW0	Non Coh. Gain SWII	Total Gain SWII
2	6 dB	0.0 dB	6.0 dB	0.0 dB	6.0 dB
3	6 dB	2.65 dB	8.65 dB	3.8 dB	9.8 dB
4	6 dB	4.4 dB	10.4 dB	6 dB	12.0 dB

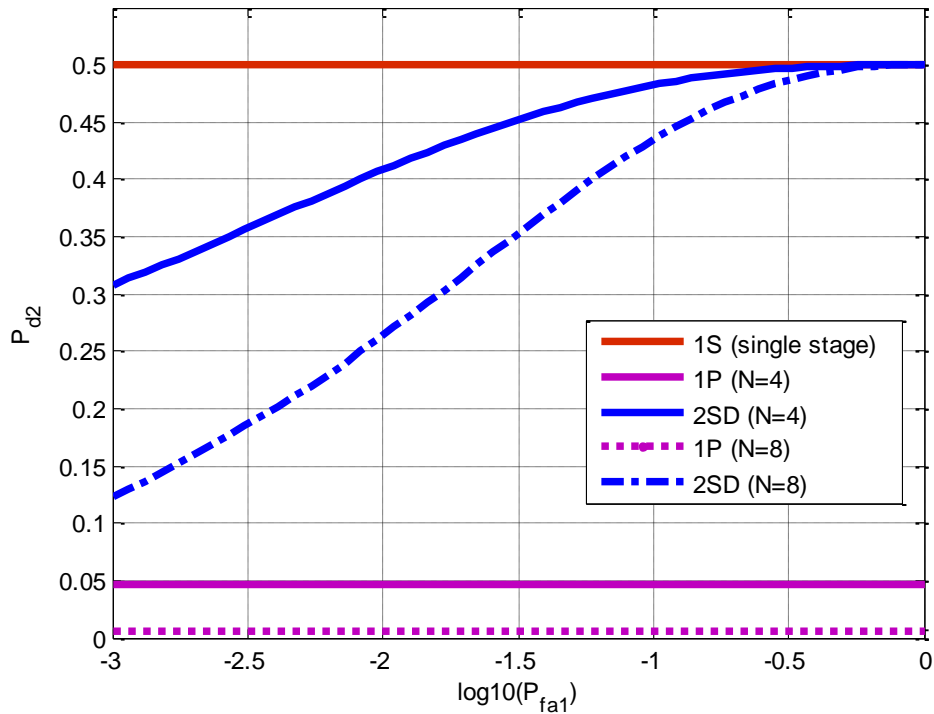
* Non Coherent Gain Calculated for PD=0.9 PFA=10e⁻⁶. Gain w.r.t. single monostatic radar with 4 pulse NCI
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Results: Overview

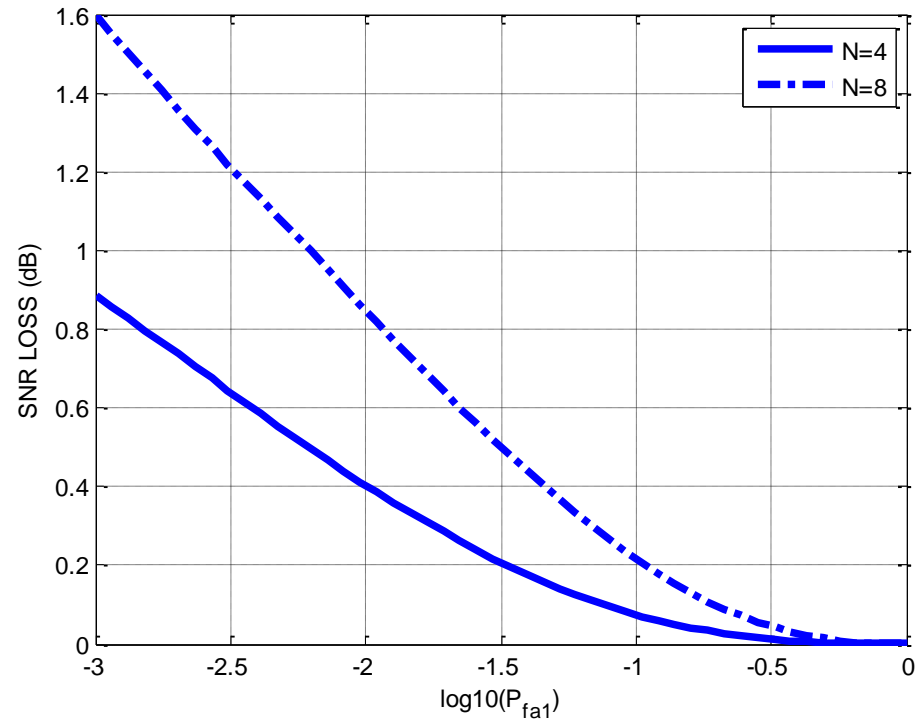
- In the following set of slides we cover three different analyses of the two-step detector's (2SD) performance under the SW2 target model:
 - 1) Overall probability of detection (P_{d2}) versus the level of Stage 1 censoring (P_{fa1}).
 - 2) Overall probability of detection (P_{d2}) versus target SNR.
 - 3) Sensitivity of 2SD to assumed target SNR.
- In our analysis we consider the Cooperative Networked Radar (CNR) implementation of statistical MIMO radar for two cases:
 - I. $N = 4$ platforms with each platform transmitting $M = 1$ pulse (for a total of 16 independent pulses), overall probability of false alarm (P_{fa2}) is 10^{-6} .
 - II. $N = 8$ platforms with each platform transmitting $M = 1$ pulse (for a total of 64 independent pulses), overall probability of false alarm (P_{fa2}) is 10^{-6} .
- In this analysis the target SNR is selected based on a P_d of 0.5 for the un-censored case ($P_{fa1} = 1$: single-stage detector)

Results: (1) P_{d2} vs. P_{fa1}

- Target SNRs are 2.4 dB and -1.4 dB per pulse per platform for the $N = 4$ and 8 cases respectively.



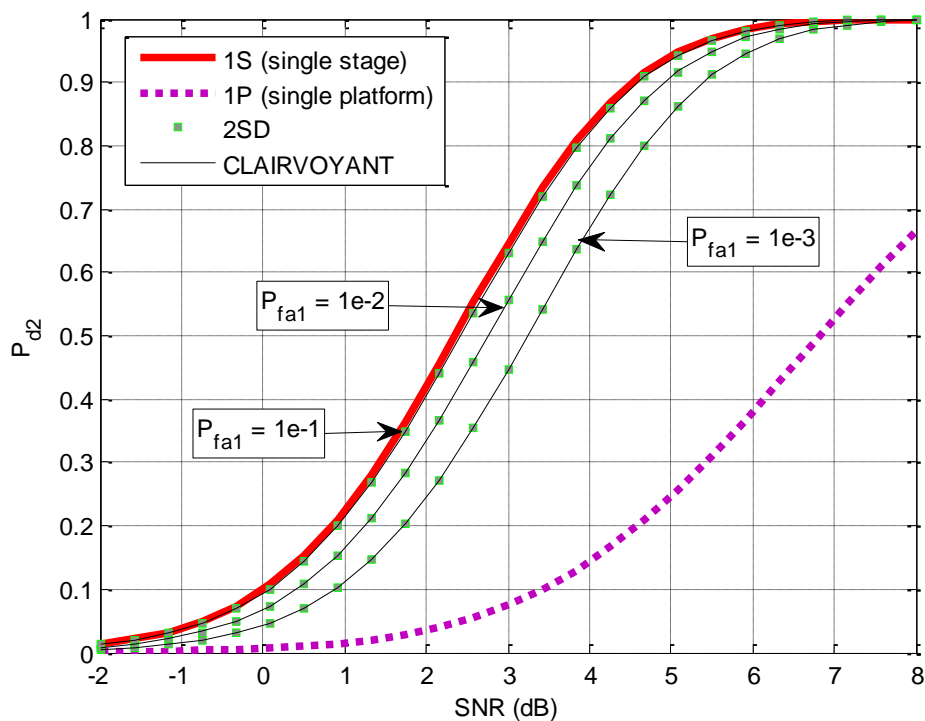
P_{d2} vs. P_{fa1}



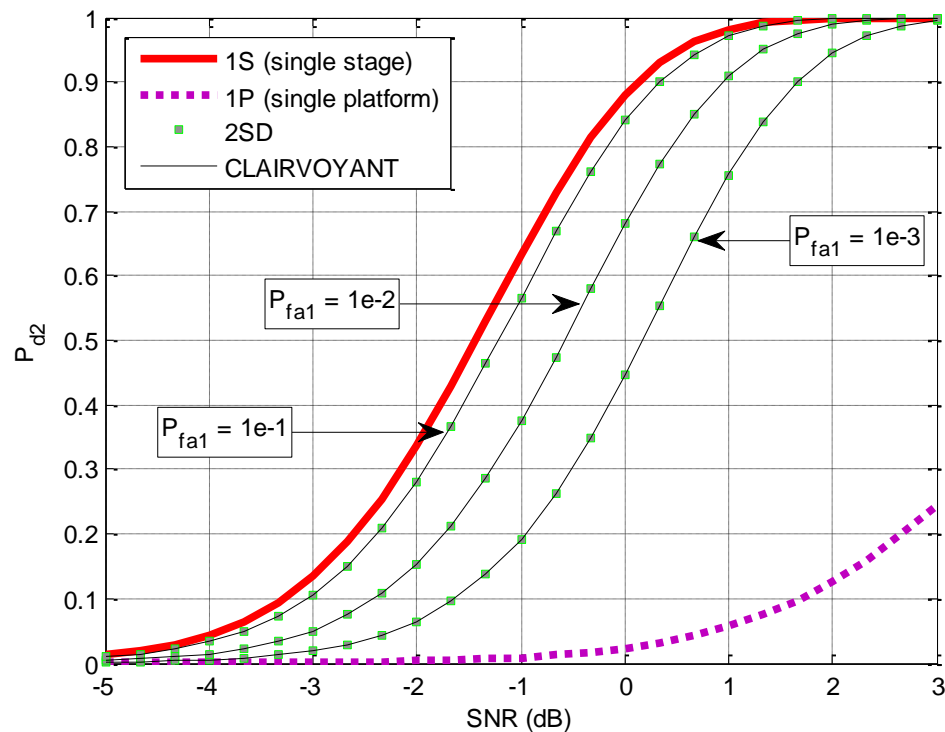
Equivalent SNR Loss

Results: (2) P_{d2} vs. SNR

- In this analysis we contrast the fully-clairvoyant 2SD with a practical 2SD for three different values of P_{fa1} .
- For the practical 2SD we use an assumed target SNR in order to select the target thresholds with the SNRs selected to be the same as the previous slide (2.4 dB and -1.4 dB per pulse per platform for the $N = 4$ and 8 cases respectively).



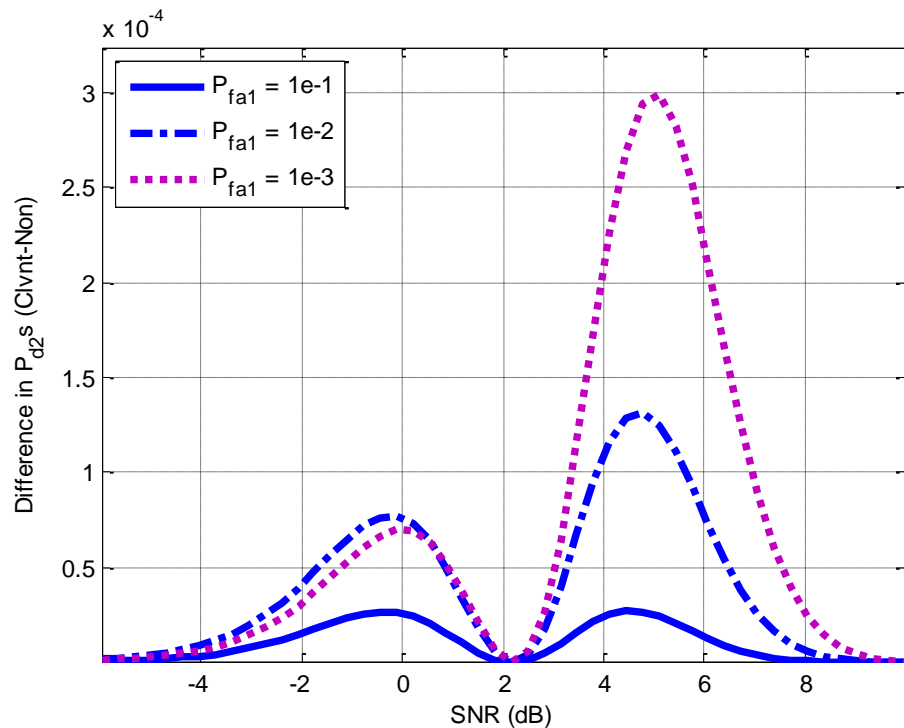
$N = 4$



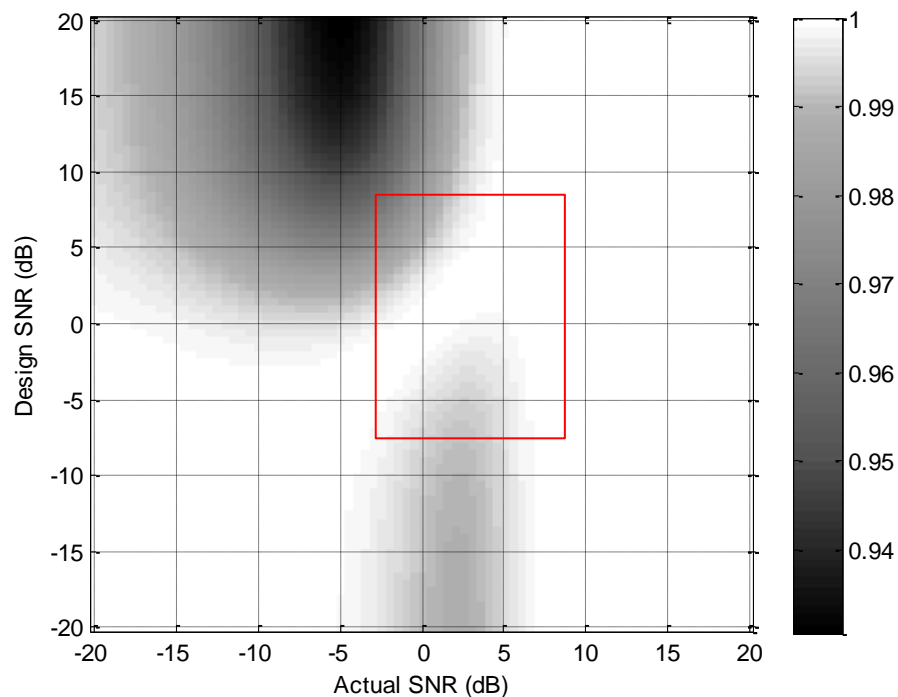
$N = 8$

Results: (3) Sensitivity to Assumed SNR

- It is clear from the previous slide that the performance of the clairvoyant and practical 2SDs are almost identical.
- Empirical results for $N = 4$ case:



Delta between
Clairvoyant P_{d2} and Practical P_{d2}



Ratio of Practical P_{d2}
to Clairvoyant P_{d2}
($P_{fa1} = 10^{-3}$)

Conclusions

- Examined the two-step detection scheme that arises when practical data-rate limits are imposed on a distributed detection system.
- Derived the Neyman-Pearson two-step detection rule in the general case and in the case when the underlying target is modeled as Swerling 2 (fluctuating RCS).
- Formulated closed-form expressions for the second-stage probability of false alarm and probability of detection.
- Shown that the thresholds can be easily selected using a root-finding method and an assumed target SNR.
- We have illustrated the performance of the two-step detector for the $N=4$ and $N=8$ Cooperative Networked Radar cases.
- Shown that the NP two-step detection rule for the Swerling 2 target model is clairvoyant and have provided some empirical evidence it is only weakly dependent on the assumed target SNR.